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Linewidth and Dispersion of the Virtual Magnon Surface State in Thick Ferromagnetic Films

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The magnetostatic surface state in thick ferromagnetic films with an applied magnetic field H parallel to the surface is shown to be a virtual surface state when both exchange and dipolar interactions are included. Simple expressions for the frequency as a function of the wave vector parallel to the surface $\Omega(k_y)$ and the effective linewidth ΔH (due to decay into bulk magnons) of the virtual surface state are calculated by power-series expansion of the exact eigenvalue equation. The width ΔH increases with increasing applied field H and is proportional to k_y^3. For an yttrium iron garnet film with H on the order of 10^3Oe, $\Delta H \lesssim 40$ Oe for $k_y \leq 10^5\,\text{cm}^{-1}$. For $k_y \sim 10^6\,\text{cm}^{-1}$, ΔH becomes too large to employ the employ the virtual-state concept and no well-defined surface state exists.

It was shown by Damon and Eshbach[1] that in the absence of the exchange interaction a magnetostatic surface spin-wave state would exist in a ferromagnetic film when the applied and internal fields were parallel to the surface.

A number of recent studies[2–8] have addressed the problem of determining the effect of exchange on the Damon-Eshbach (DE) surface state. The effects of exchange on the bulk and DE surface state in *thin* films have been discussed by Wolfram and De Wames.[2–5] In thin films where the frequency spacing between bulk branches is much larger than the bulk magnon linewidth, the surface branch interacts with an individual bulk exchange branch when the two branches are near the crossing point. The surface branch is split into segments which join neighboring exchange branches.[2,3]

In very thick films the bulk magnon branches overlap in frequency and form a continuous bulk spectrum. *A surface branch cannot exist within a continuum of bulk states*. This means that the DE surface state must be regarded as a virtual surface state with a width due to losses into the bulk continuum of magnon states.

In this paper we report the calculation of the dispersion $\Omega(k_y)$, linewidth ΔH, and the range of wave vectors k_y parallel to the film surface for which the virtual magnetic surface state exists. The linewidth ΔH increases with the applied field H and is proportional to the k_y^3. For yttrium iron garnet (YIG), $\Delta H \lesssim 40$ Oe for an applied field H parallel to the surface of the order 10^3 Oe for $k_y \leq 10^5$ cm^{-1}. No sharp cutoff exists. The linewidth increases smoothly with k_y until for $k_y \sim 10^6$ cm^{-1} the virtual-state concept breaks down and no well-defined state exists for YIG.

In very thick films the back surface is less important for the surface wave and the film may be approximated by a ferromagnetic half-space extending to the right (positive x) of the $x=0$ plane. The magnetization M_0, internal field H_i, and applied field H are parallel to the z axis. Spin waves with wavelengths long compared to the atomic spacing are solutions of Maxwell's equations and the equations of motion for the magnetization:

$$\nabla \times \vec{h} = \nabla \cdot (\vec{h} + 4\pi\vec{m}) = 0 \quad , \tag{1}$$

$$-i\omega\vec{m} = \gamma\vec{M}_0 \times \{h - [(H_i/M_0) - D\nabla^2]\vec{m}\} \quad . \tag{2}$$

In Eqs. (1) and (2), the quantities h and m are the small transverse components of the magnetic field and magnetization that vary in time as $e^{-i\omega t}$ and D is the exchange constant. If we introduce a scalar magnetic potential ψ such that $-\nabla\psi = \vec{h}$, then Eqs. (1) and (2) give a sixth-order differential equation[9]

$$\left((\Omega^2 - O^2 - O)\nabla^2 + O\frac{\partial^2}{\partial z^2}\right)\psi = 0 \quad . \tag{3}$$

The operator O is $\Omega_H - (D/4\pi)\nabla^2$ and the dimensionless quantities Ω and Ω_H are, respectively, $\omega/4\pi\gamma M_0$ and $H_i/4\pi M_0$. For simplicity, we consider here only waves propagating along the y axis (perpendicular to the applied field). In general, the solutions in the ferromagnetic medium are suppositions of waves with three different wave vectors.[2] For the case considered here, the solution is[10]

$$\psi = C\exp(ik_y y)\exp(|k_y|x) \quad , \quad x \leq 0$$
$$= \sum_{l=1}^{3} A_l \exp(ik_y y)\exp(ik_l x) \quad , \quad x \geq 0 \quad . \tag{4}$$

The quantities $k_l (l = 1, 2, \text{ and } 3)$ are

$$k_1 = (4\pi/D)^{1/2}[(\Omega^2 + \tfrac{1}{4})^{1/2} - (\Omega_k + \tfrac{1}{2})]^{1/2} \quad ,$$
$$k_3 = i(4\pi/D)^{1/2}[(\Omega^2 + \tfrac{1}{4})^{1/2} + (\Omega_k + \tfrac{1}{2})]^{1/2} \quad , \tag{5}$$
$$k_2 = ik_y \, ,$$

where $\Omega_k = \Omega_H + (D/4\pi)k_y^2$. The quantity k_2 produces a DE surface wave and is *independent of the ex-*

change parameter. The k_1 wave is a propagating sinusoidal wave that dominates the bulk states. The k_3 wave is an attenuated wave and has no analog in the magnetostatic limit (i.e., $D=0$). The eigenstates of the system are admixtures of these three types of waves. In the magnetostatic limit, the quantity k_1 is determined by a limiting process in which $(\Omega^2+\frac{1}{4})^{1/2} \to (\Omega_k+\frac{1}{2})$ as $D\to 0$. When $D=0$, the eigenstates are pure-bulk (k_1 waves) or pure-surface (k_2 waves) waves.[1] When the exchange interaction is included, a pure-surface wave is *not* possible.

The bottom of the bulk spin-wave continuum Ω_B is defined by $\Omega_B=(\Omega_k^2+\Omega_k)^{1/2}$. For $\Omega>\Omega_B$ the quantity k_1 is real, but when $\Omega<\Omega_B$ then k_1 is a (positive) imaginary number.

The constants C and A_I are determined from the boundary conditions.[11] The quantities h_y and $h_x+4\pi m_x$ are continuous across the $x=0$ plane and in this calculation the normal derivatives of m, $\partial m_x/\partial x$ and $\partial m_y/\partial x$ vanish[6,9] at $x=0$. The requirement that the determinant of the coefficients vanish leads to the exact eigenvalue condition,

$$-2(2\Omega_H-2\Omega+1)k_1k_3[\Omega k_y(k_1-k_3)+i(\Omega^2+\tfrac{1}{4})^{1/2}(k_y^2+k_1k_3)]+k_y^2[k_y(k_1-k_3)+i(k_1^2-k_3^2)]=0 \quad . \qquad (6)$$

For $\Omega>\Omega_B$ there are *no* solutions of Eq. (6) for real Ω. This follows from our previous arguments since the bulk spin-wave spectrum for the magnetic half-space is a continuum extending upwards in frequency from Ω_B as indicated schematically in Fig. 1. A surface branch cannot be degenerate with the bulk modes.

A physical interpretation of the above results can be given. In a thick sample, the discrete bulk branches overlap whenever the spacing between levels is smaller than the level widths due to all scattering and loss mechanisms. As the sample size increases, the spacing between levels decreases so that in very thick films only a very small width is required to smear the bulk spectrum into a continuous spectrum. In the thick-film limit, the bulk wave scattering and loss mechanisms required for a continuous spectrum are arbitrarily small. Approximating a thick film by a half-space is equivalent to neglecting reflections from the back surface. This implies that the energy of a reflected wave is dissipated into the crystal and eventually adds an infinitesimal increment of heat to the sample.

The exchange interaction effectively couples a surface wave to the bulk waves degenerate with it because of scattering at the $x=0$ surface. The admixture of bulk and surface waves making up the eigenstates is determined by the boundary conditions. A surface excitation will lose energy into the bulk continuum states which are degenerate with it. The rate of decay of an excitation is proportional to the imaginary part of the frequency so that solutions of Eq. (6) with real frequencies cannot be found.

Solutions of Eq. (6) can be found for complex $\Omega_S=\Omega_R+i\Omega_I$ and correspond to virtual states with finite lifetimes. Such solutions are physically meaningful if $|\Omega_I/\Omega_R|$ is small. The virtual DE surface-state dispersion and width can be found by expansion of Eq. (6) in powers of the parameter $\kappa=(D/4\pi)^{1/2}k_y$. Neglecting fourth-order terms in κ leads to the result that

$$\Omega_R=\Omega_0+2\kappa^2-|\kappa_3|(1-8\kappa_1^2\Omega_0)(\Omega_0^2+\tfrac{1}{4})^{-1/2}\kappa^3, \qquad (7)$$

$$\Omega_I=-\kappa_1(1+8|\kappa_3|^2\Omega_0)(\Omega_0^2+\tfrac{1}{4})^{-1/2}\kappa^3 \quad . \qquad (8)$$

In Eqs. (7) and (8), $\Omega_0=\Omega_H+\frac{1}{2}$ and the quantities $\kappa_1=(D/4\pi)^{1/2}k_1$ and $\kappa_3=(D/4\pi)^{1/2}k_3$ are evaluated at Ω_0. The quantity Ω_I is always negative and corresponds to a decay with positive time, since the fields vary as $e^{-i\omega t}$. The admixture of k_1 and k_3 waves into the surface state is proportional to k_y^2

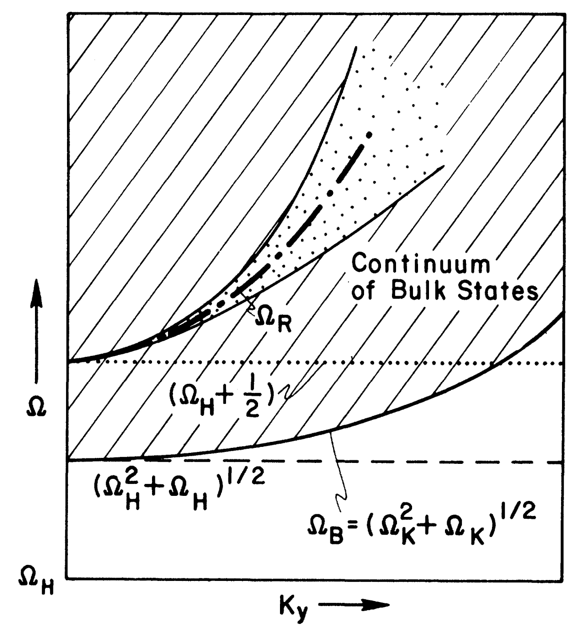

FIG. 1. Schematic of the spectrum of a magnetic half-space. The cross-hatched region indicates the continuum of bulk states above Ω_B the band edge. The dotted line indicates the position of the surface state for the exchange interaction $D=0$. The dashed curve indicates the position of all of the bulk spin-wave branches when $D=0$. The dot-dashed curve is Ω_R for the virtual surface state with $D\neq 0$. The shaded portion indicates the width of the virtual state or the range over which it mixes with the bulk states.

for small k_y. For large H, the quantities Ω_R and Ω_I approach the values

$$\Omega_R = \Omega_0 + 2\kappa^2 - (\sqrt{2}/16)\Omega_0^{-5/2}\kappa^3 \quad , \tag{9}$$

$$\Omega_I = -4(2\Omega_0)^{1/2}\kappa^3 \tag{10}$$

For $\kappa \ll 1$, Eqs. (9) and (10) are accurate to a few percent for all values of H including $H = 0$. The effective linewidth ΔH is $4\pi M_0 \Omega_I$. For YIG, $4\pi M_0 = 1750$ Oe, $D/4\pi \approx 2.6\times 10^{-12}$ cm^2, so that if $H = 10^3$ Oe and $k_y = 6\times 10^4$ cm^{-1}, then $\kappa \approx 0.1$, and we find that $\omega/\gamma = 4\pi M_0 \Omega_R \approx 1.9\times 10^3$ Oe while $\Delta H \sim 10$ Oe. Increasing k_y to 10^5 cm^{-1} increases ΔH to about 40 Oe. For $k_y \sim 3.5\times 10^5$ cm^{-1}, $\Delta H \sim 4\pi M_0$ and the concept of the virtual state begins to break down. Decreasing H decreases ΔH at most by a factor of $\sqrt{2}$. Thus for YIG with fields H less than or on the order of a few kG, Ω_I will exceed Ω_R in the range 10^5 cm$^{-1} < k_y < 10^6$ cm^{-1}. The latter value of k_y is an *upper bound for the existence of a* DE *state in* YIG.[12]

The cubic term in Ω_R may be associated with the interaction between a bulk magnon branch and the surface branch near crossing in thin films.[2]

Below the spin-wave band ($\Omega < \Omega_B$), all three of the k_l are imaginary. An exact solution of Eq. (6) is obtained for $\Omega = (\Omega_H^2 + \Omega_H)^{1/2}$ independent of k_y. For this value of Ω, $k_1 = ik_y$. The coefficients A_l corresponding to this solution are $A_1 = -A_2$ and $A_3 = 0$, so that according to Eq. (4) *the magnetic potential* ψ *vanishes at all points in space*. Therefore this solution is spurious and does not correspond to a physical state of the system.

Studies of the magnetic half-space based on a variational approximation have been reported by Benson and Mills.[6] They did not obtain a lifetime for their approximate surface state, but they did obtain a *second-order correction* to Ω_R which agrees with that of Eq. (7). They also reported that there existed a critical value of k_y, $k_c \approx 6\times 10^4$ cm^{-1} above which no surface-state solutions existed. The present calculations show that no sharp cutoff exists. The width of the surface state is only 10 Oe at k_c, and grows *smoothly* with increasing k_y.

A second surface branch reported by Benson and Mills[6] has a wave-vector frequency relation not permitted by Eq. (5). The k_l given in Eq. (5) are the only ones which satisfy the requirements of Maxwell's equations in the ferromagnetic medium, and consequently we cannot obtain solutions from Eq. (6) of the type described by Benson and Mills.

In contrast to the results of Carr *et al.*,[8] we find no solutions with frequencies below the spin-wave band.

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[1] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids, 19, 308 (1960).

[2] T. Wolfram and R. E. De Wames, Solid State Commun. (to be published).

[3] T. Wolfram and R. E. De Wames, Phys. Letters 30A, 3 (1969).

[4] T. Wolfram and R. E. De Wames, North American Rockwell Corporation Technical Report No. SCTR-69-22, 1969 (unpublished).

[5] R. E. De Wames and T. Wolfram, J. Appl. Phys. (to be published).

[6] H. Benson and D. L. Mills, Phys. Rev. Letters 23, A8 (1969).

[7] H. Benson and D. L. Mills, Phys. Rev. 178, 839 (1969).

[8] P. H. Carr, A. J. Slobodnick, Jr., and J. C. Sethares (unpublished).

[9] Equation (3) is conveniently obtained by calculating $\nabla\cdot m$ and $\nabla\times m$ from Eq. (2). The quantity $\nabla\times m$ can then be expressed in terms of $\nabla\cdot m = (1/4\pi)\nabla^2\psi$. This leads to the general expression

$$\left[(\Omega^2 - O^2 - O)\nabla^2 + O\left(\frac{M_0\cdot\nabla}{M_0}\right)^2\right]\psi = 0 \quad .$$

[10] Using Eq. (2), the magnetization may be written as be written as

$$m = e^{ik_y y}\sum_{l=1}^{3} m_l e^{ik_l x} \quad ,$$

where $m_l = m_l(A_1, A_2, A_3, C)$ and the constants A_l and C are determined by the boundary conditions.

[11] Boundary conditions are discussed in the following references: W. S. Ament and G. T. Rado, Phys. Rev. 97, 1558 (1955); P. Pincus, *ibid.* 118, 658 (1960); A. I. Akhiezer, V. G. Bar'yakhtar, and M. I. Kaganov, Usp. Fiz. Nauk 71, 533 (1960) [Soviet Phys. Usp. 3, 567 (1961)]; F. R. Morgenthalar, MIT Tech Report No. 14, 1967 (unpublished).

[12] This result does not depend upon the specific boundary conditions employed here. Boundary conditions other than the vanishing of the normal derivatives of m have been studied and lead to similar results for Ω_S.